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The *minus* sign indicates merely difference in these expressions, thus avoiding negative terms. In some of the calculations the differences are but a repetition of the sums.

To continue the series, the value of  $m$  is the next succeeding value of  $n$ , and the value of  $m+n$  is the next succeeding value of  $\sqrt{(2n^2 \pm d)}$ .

Take  $d=1=2 \times 1^2 - 1^2$ . Then  $r=1$  and  $s=1$ . Whence,  $n=1, 2, 5, 12, 29$ , etc.;  $m=2, 5, 12, 29, 70$ , etc.

Substituting these values in  $(2mn)^2 + (m^2 - n^2)^2 = (m^2 + n^2)^2$ , we find the following right triangles in which the difference of the legs is 1: 4, 3, 5; 20, 21, 29; 120, 119, 169; 696, 697, 985; 4060, 4059, 5741; etc.

Take  $d=7=2 \times 2^2 - 1^2$ . Then  $r=2$  and  $s=1$ . Whence  $n=2, 1, 3, 4, 8, 9, 19, 22$ , etc.;  $m=3, 4, 8, 9, 19, 22, 46, 53$ , etc.

The right triangles are 12, 5, 13; 8, 15, 17; 48, 55, 73; 72, 65, 97; 304, 297, 425; 396, 403, 565; etc.

The sides of another set of triangles will be 7 times the sides of those the difference of whose legs is 1; as, 28, 21, 35; 140, 147, 203; 840, 833, 1183; etc.

#### REMARK ON PROBLEM 98, BY CHARLES C. CROSS, WHALEYVILLE, VA.

The solution given by Dr. Drummond of the second part of this problem does not appear to me to satisfy all the required conditions.

Let  $x$  and  $y$  be the numbers. Then

$$\begin{aligned} x+1 &= \square = a^2 \text{ (say)} \dots (1); & y+1 &= \square = b^2 \text{ (say)} \dots (2); \\ x+y+1 &= \square = c^2 \text{ (say)} \dots (3); & x-y+1 &= \square = d^2 \text{ (say)} \dots (4). \end{aligned}$$

(1) and (2) in (3) and (4) give  $a^2 + b^2 - 1 = c^2$  and  $a^2 - b^2 + 1 = d^2$ ; adding  $2a^2 = c^2 + d^2$ . Let  $c = m+n$  and  $d = m-n$ , then  $a^2 = m^2 + n^2$ . Let  $a^2 = (p^2 + q^2)^2$ ,  $m^2 = (p^2 - q^2)^2$ , and  $n^2 = (2pq)^2$ . Then  $c = p^2 - q^2 + 2pq$ , and  $d = p^2 - q^2 - 2pq$ . Therefore  $x = (p^2 + q^2)^2 - 1$ , and  $y = 4pq(p^2 - q^2)$ .

In order that this value of  $y$  may satisfy the conditions of the problem,  $p^2 - q^2$  must equal  $pq \pm 1$ . Whence  $q = [\sqrt{(5p^2 \pm 4)} - p]/2$  in which  $5p^2 \pm 4$  is to be made a square.

Let  $p=2$ , then  $q=1$ .  $\therefore x=24$ ,  $y=24$ .

Let  $p=3$ , then  $q=2$ .  $\therefore x=168$ ,  $y=120$ .

Whence  $x+1=13^2$ ;  $y+1=11^2$ ;  $x+y+1=17^2$ ; and  $x-y+1=7^2$ . And so on for other values of  $p$ .

#### 101. Proposed by HARRY S. VANDIVER, Bala, Pa.

Prove that it is impossible to find integral values for  $x$ ,  $y$ , and  $z$  such that the relation  $x^2y + xz^2 = y^2z$  is satisfied.

Solution by the PROPOSER.

Suppose that  $x$ ,  $y$ , and  $z$  are integers that satisfy  $x^2y + xz^2 = y^2z$ ....(1), then we find

$$\begin{vmatrix} x & y & -z \\ y & -z & x \\ -z & x & y \end{vmatrix}^3 = \begin{vmatrix} X & Y & Z \\ Y & Z & X \\ Z & X & Y \end{vmatrix} \dots (2),$$

where

$$\begin{aligned} X &= x^3 + y^3 - z^3 - 3xyz \\ Y &= 3(x^2y + xz^2 - y^2z) = 0. \\ Z &= 3(xy^2 + yz^2 - x^2z) \end{aligned}$$

Then substituting in (2) and expanding

$$X^3 + Z^3 = (x^3 + y^3 - z^3 + 3xyz)^3$$

which is impossible, since the sum of two integral cubes cannot be an integral cube. (For a proof, see Euler's *Algebra*.) Hence the impossibility of (1) is established.

Also solved by the late **JOSIAH H. DRUMMOND**.

#### AVERAGE AND PROBABILITY.

**122.** Proposed by F. M. PRIEST, St. Louis, Mo.

Suppose each of the nine digits to be placed in a wheel, and five of them drawn at random therefrom, and written down in the order drawn. What is the probability the number thus expressed will be greater than 50,000?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and J. F. LAWRENCE, A. B., Breckenridge, Mo.

If 5, 6, 7, 8, or 9 be drawn first, the number will be greater than 50,000.

The chance of drawing 5 is  $\frac{1}{9}$ ; of drawing 6,  $\frac{1}{9}$ ; of drawing 7,  $\frac{1}{9}$ ; of drawing 8,  $\frac{1}{9}$ ; of drawing 9,  $\frac{1}{9}$ . The chance of drawing 5, 6, 7, 8, or 9 is, therefore,  $\frac{5}{9}$ . Therefore the chance that the number is greater than 50,000 is  $\frac{5}{9}$ .

**123.** Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Three points are taken at random within a square. What is the probability that the angle formed by joining them is acute?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let  $ABCD$  be the square;  $P$ ,  $Q$ ,  $R$  the three random points. Through  $P$ ,  $Q$  draw  $GH$ , and through  $Q$  draw  $KL$  perpendicular to  $GH$ . When  $P$  is between  $G$  and  $Q$  the angle  $PQR$  will be obtuse if  $R$  lies on the opposite side of  $LK$  from  $P$ . Draw  $AF$  perpendicular to  $KL$ ;  $AES$  perpendicular to  $GH$ ;  $ST$  parallel to  $GH$ ;  $MN$  and  $TU$  parallel to  $KL$ ;  $DM$  parallel to  $GH$ .

Let  $AB=a$ ,  $PQ=x$ ,  $AF=y$ ,  $AE=z$ ,  $\angle KAF=\theta$ . Then  $AK=y\sec\theta$ ,  $BK=$

